

# Fluctuations in Hadronizing QGP

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The dynamical development of the cooling and hadronizing quark-gluon Plasma (QGP) is studied in a simple model assuming critical fluctuations in the QGP to Hadronic Matter (HM) and a first order transition in a small finite system. We consider an earlier determined free-energy density curve in the neighbourhood of the critical point, with two local minima corresponding to the equilibrium hadronic and QGP configurations. In this approach the divergence at  $e = 0$  eliminates fluctuations with negative or zero energy. The barrier between the equilibrium states is obtained from an estimated value of the surface tension between the two phases. We obtain a characteristic behavior for the skewness and the kurtosis of energy density fluctuations, which can be studied via a beam energy scan program.

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## I. INTRODUCTION

In central heavy ion collisions fluctuations may occur due to critical phenomena arising from a phase transition in the Equation of State (EoS). Fluctuations arising from the initial asymmetry are smaller in these head-on reactions. In the neighborhood of the critical point the shear viscosity of the quark-gluon Plasma (QGP) is becoming small [1], which also facilitates the appearance of fluctuations.

The flow effects depend on the initial state profile and on the viscosity. Turbulence appears only for small viscosity, which indicates the critical point of the matter [1], and it is a sensitive measure of viscosity and its minimum at the critical point.

Molecular dynamics and Fluid Dynamics simulations of heavy ion collisions suggest that collective flow asymmetries can be measured [2], both if the global asymmetry or random flow arising from the initial state transverse momentum fluctuations or longitudinal center of mass rapidity,  $y_{CM}$ , fluctuations are causing it. These alternative sources may also lead to specific statistical characteristics as discussed in ref. [3]. These phenomena contribute to a spatial spread of the matter and energy density variations are present even if we do not have a phase transition in our EoS.

Here, on the other hand, we study random fluctuations of thermodynamical origin caused by the phase transition based on the considerations described in ref. [4]. The field is under intensive recent study, and several aspects of the phase transitions have discussed the non-Gaussian fluctuations [5].

For a realistic reaction model we have to describe the final stage of the reaction also. In order to study the properties of the phase transition we need to have sufficient energy to form QGP in a sufficiently large volume, and then the system must hadronize. At relatively low energies this hadronization might happen well before the freeze out (FO) stage, and then we have only rather in-

direct information about the phase transition.

In our recent studies [3] we use a Multi Module Model or Hybrid Model approach to describe high energy heavy ion collisions in the RHIC and LHC energy range. Then from the locally equilibrated QGP we have to form hadrons. We do not assume that the hadronization happens in chemical equilibrium as this would take too long time [6] and would not allow for baryons of high strangeness.

In high energy heavy ion reactions around 50 GeV/nucleon or above when QGP is formed, we have low mass quarks (5-10 MeV) and massless gluons in large numbers and the conserved baryon charge has a minor effect. On the hadronic side also mainly mesons and baryon-antibaryon pairs are formed. The observable phase transition happens when the plasma expands, hadronizes, and freezes out. At this stage the system may be in the vicinity of the critical point so critical fluctuations may appear. In a finite volume this would show up as energy density fluctuation, which would then lead to charged hadron number fluctuation also (and much less in net baryon charge fluctuations).

In the present simple model we assume two coexisting phases in a finite system near a first order phase transition. We study the abundance of the energy density distribution of the two phases in the mixed phase domain in terms of the volume ratio.

This dynamical development is not directly observable, nevertheless, we can observe the FO moment, which should be similar for different locations in the system.

At the same time we can vary the beam energy, which shifts the FO point versus the critical point, so a beam energy scan can provide us the series of information we are interested in.

## II. CRITICAL FLUCTUATIONS

Following ref. [4] we briefly present the way to describe critical fluctuations following the ideas of the Landau-Ginsburg theory for critical phenomena.

In case of QGP to hadronic matter (HM) transition the essential difference is that in phase equilibrium the energy density of the HM phase is much lower than that of the QGP phase. As a consequence the usual 4th order polynomial to describe the free energy of the system is not realistic as it would lead to considerable population of negative energy density states, which would be unphysical.

This problem was solved in ref. [4], where the 4th order polynomial approach was substituted by a Laurent series.

Using the simple bag model EoS the equilibrium value of the energy density in the low-temperature, low-energy-density phase (HM) denoted by  $e_h$ , and the equilibrium value of the energy-density in the high-temperature, high-energy-density, QGP, phase denoted by  $e_q$  can be calculated as

$$e_h(T) = \frac{\pi^2}{10(\hbar c)^3} T^4 \quad (1)$$

and

$$e_q(T) = \frac{\pi^2}{(\hbar c)^3} \left( \frac{37}{30} T^4 + \frac{34}{90} T_c^4 \right). \quad (2)$$

To study the fluctuations we find the free energy,  $f(e)$ , for arbitrary values of  $e$ , and not just for the energy densities  $e_q$  and  $e_h$ . For these equilibrium points:

$$f(e_q(T)) = -p_q(T) = -\frac{\pi^2}{90(\hbar c)^3} (37T^4 - 34T_c^4), \quad (3)$$

$$f(e_h(T)) = -p_h(T) = -\frac{\pi^2}{30(\hbar c)^3} T^4. \quad (4)$$

Following the Landau theory we approximate now the free energy density as a polynomial in the neighborhood of an  $e_0$  energy density ( $e_0 \in [e_h, e_q]$ ), where it has a local maximum. In order to obtain the required divergence at  $e = 0$ , a slightly modified functional form is assumed:

$$f(e) = f_1 + \frac{K_1}{e} + K_2(e - e_0) + K_3(e - e_0)^2 + K_4(e - e_0)^3. \quad (5)$$

The constants,  $f_1, K_1, K_2, K_3, K_4$  can be determined from thermodynamic considerations as done in ref. [4]. One will obtain

$$K_1 = \frac{\sigma}{\xi_0} \frac{1}{A_0 A_1}, \quad (6)$$

$$K_2 = \frac{K_1}{e_0^2}, \quad (7)$$

$$K_3 = -\frac{K_1}{2} \frac{e_h^2(e_q + e_0) + e_q^2(e_0 + e_h) - e_0^2(e_q + e_h)}{e_q^2 e_h^2 e_0^2}, \quad (8)$$

$$K_4 = \frac{K_3}{3} \frac{e_h e_q + e_q e_0 + e_0 e_h}{e_q^2 e_h^2 e_0^2}, \quad (9)$$

where

$$A_0 = \frac{(e_q - e_0)(e_0 - e_h)}{e_0^3 e_h^2 e_q^2} \cdot (2e_q e_h + e_0 e_q + e_h e_0), \quad (10)$$

$$A_1 = \frac{(e_q - e_h)^2}{3\sqrt{2}} + \frac{[e_0 - (e_q + e_h)/2]^2 \sqrt{\pi}}{2}. \quad (11)$$

In Eq. (6)  $\sigma$  represents the surface tension of a hadronic bubble and  $\xi_0$  is the characteristic size of a hadronic droplet. Once the  $K_i$  values are expressed as a function of  $e, e_0, e_q, e_h, \xi_0, \sigma$  values, the unknown  $f_1$  and  $e_0$  parameters are obtainable at any temperatures from Eqs (3) and (4). Following our previous work [4], we have used the  $T_c = 0.169 \text{ MeV}$ ,  $\xi_0 = 3 \text{ fm}$  and  $\sigma = 0.05 \text{ GeV/fm}^2$  values in all our calculations.

For temperatures in the vicinity of the critical temperature the free energy density curve as a function of the energy density is illustrated in Fig. 1.

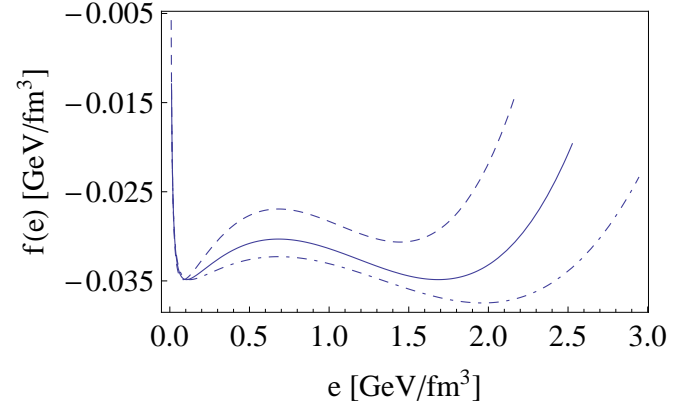


FIG. 1: (color online) The free energy density as a function of energy density  $e$ , for  $T = 0.95T_c$  (dashed),  $T = T_c$  (full line) and  $T = 1.05T_c$  (dot-dashed).

Once the free energy curve is known, one can estimate the probability density of finding the system in a state with energy density  $e$ :  $P(e) \propto \exp(-\beta F(e))$ , where  $F(e) = \Omega f(e)$ , with  $\Omega$  the volume of the created QGP. On Fig. 2 we plot at the critical temperature the characteristic  $P(e)/P(e_q)$  curves, considering different volumes for the QGP ( $\Omega = 10 \text{ fm}^3$ ,  $50 \text{ fm}^3$ , and  $500 \text{ fm}^3$  values). Also, in Fig. 3 we show the  $P(e)/P(e_q)$  curves for  $\Omega = 500 \text{ fm}^3$  and the same temperatures as in Fig. 1.

It is also important to mention that different thermodynamical parameters (especially intensives and extensive ones) do not have to show the same critical fluctuation properties, so we have to study the fluctuations of several parameters. Furthermore, the statistical physics

estimates assume a single thermal source at or near the critical point, while we estimate here also the effects of spatial fluctuations, which arise from a dynamically expanding fluid flow even in the least fluctuating configuration.

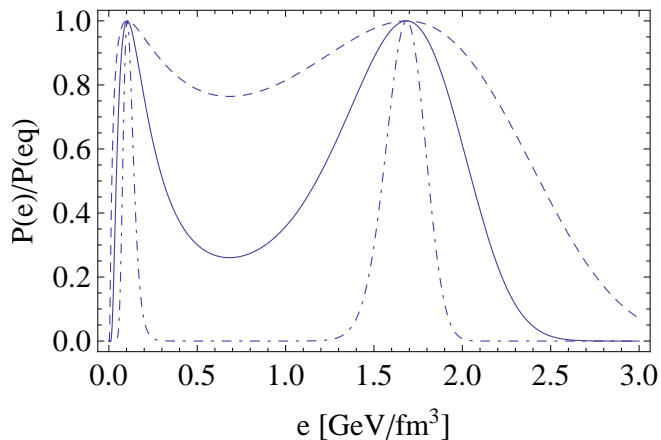


FIG. 2: (color online) The relative probability of finding a state of a given energy density  $e$  for  $T = T_c$ , in a system of volume  $\Omega = 10 \text{ fm}^3$  (dashed),  $\Omega = 50 \text{ fm}^3$  (full line) and  $\Omega = 500 \text{ fm}^3$  (dot-dashed).

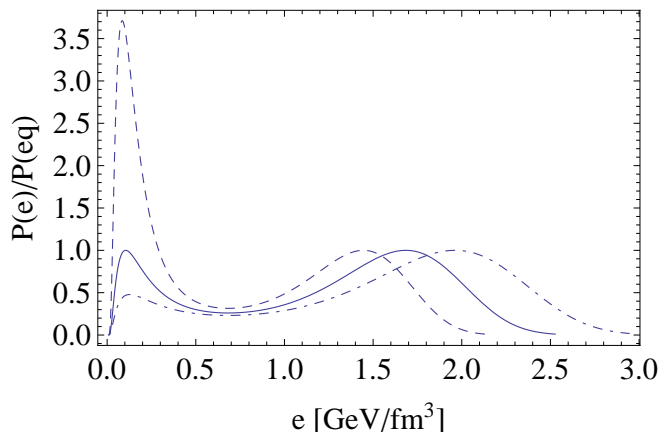


FIG. 3: (color online) The relative probability of finding a state of a given energy density  $e$  for the  $T = 0.95T_c$  (dashed),  $T = T_c$  (continuous) and  $T = 1.05T_c$  (dot-dashed) temperatures. The volume of QGP is  $\Omega = 500 \text{ fm}^3$ .

### III. HADRONIZATION AND EXPANSION

The dynamically developing flow pattern leads to a spatial distribution of all thermodynamical quantities, while the system expands rapidly. Finally the super-cooled QGP can hadronize rapidly and almost simultaneously it freezes out. This final stage of the reaction can be described by a non-equilibrium model.

We assume central collisions only, to avoid the effects from azimuthal flow asymmetries and from particle emission from projectile and target residues (spectator evaporation) [2].

In a theoretical approach we can assume a spatial distribution of the  $x$  quantity. For the variable  $x$  the averages and various order moments distributions can be written as

$$\langle x^n \rangle = \int x^n P(x) dx, \quad (12)$$

$$\begin{aligned} M^{(n)} &= \langle (x - \langle x \rangle)^n \rangle = \\ &= \int (x - \langle x \rangle)^n P(x) dx, \end{aligned} \quad (13)$$

where  $P(x)$  is the spatial distribution weighted, e.g., by the baryon charge density in the center of mass frame (CF). The spatial variance, the skewness and the kurtosis can be obtained from these moments:

$$\Delta x = \langle (x - \langle x \rangle)^2 \rangle = M^{(2)}, \quad (14)$$

$$S = \frac{\langle (x - \langle x \rangle)^3 \rangle}{(\Delta x)^{3/2}} = \frac{M^{(3)}}{(M^{(2)})^{3/2}}, \quad (15)$$

$$K = \frac{\langle (x - \langle x \rangle)^4 \rangle}{(\Delta x)^2} - 3 = \frac{M^{(4)}}{(M^{(2)})^2} - 3. \quad (16)$$

By using these averages, first we can calculate specific extensives, which are governed by strict conservation laws. The total baryon charge, energy and momentum conservations are governed by the continuity equation and by the relativistic Euler equation.

$$N^{\mu}_{,\mu} = 0, \quad (17)$$

$$T^{\mu\nu}_{,\nu} = 0, \quad (18)$$

and as a consequence, the total momentum in the CF should remain zero during the development, while the average specific energy per net nucleon number

$$\langle \varepsilon^{CF} \rangle \equiv \frac{T^{00}}{N^0} = \text{const.} \quad (19)$$

should remain constant in CF.

The total baryon number,  $N_{tot}$ , is exactly conserved in the reaction. At the same time the average baryon charge density is decreasing.

### IV. SKEWNESS AND KURTOSIS

In this section we study the skewness and kurtosis of the specific energy density and consequently the charged particle densities according to Eq. (15) and Eq. (16). We determine these quantities as a function of the systems

temperature and also as a function of the volume abundance ( $r_h$ ) of the hadronic matter. We assume that in a rapid transition, where critical fluctuations dominate and the two phases are not separated these two phases are in thermal equilibrium. The temperature decreases rapidly starting from QGP where  $T > T_c$  until the hadronization completes at  $T < T_c$ . As the phases are not separated the simplest estimate for any temperature  $T$ , is that the volume abundance of the hadronic matter is:

$$r_h = \frac{P(e_h)}{P(e_q) + P(e_h)}, \quad (20)$$

where  $e_h$  and  $e_q$  are the energy densities of the pure phases defined in eqs. (1, 2), and the probability densities,  $P(e)$ , are defined in section II. This relation makes a one to one correspondence between the volume abundance and the equilibrium temperature in a rapidly expanding and cooling system during the process of a phase transition.

The skewness, Fig. 4, is first negative (indicating a longer tail on the lower energy side), then at 80 % HM ( $r_h = 0.8$ ) volume abundance it turns into positive (indicating a longer tail on the high energy side). The hadronization can be parametrized both as a function of the volume abundance of the growing hadronic phase or the decreasing equilibrium temperature of the system with critical fluctuations.

In Fig. 5, we can see that the kurtosis is positive at first, then turns to be negative (the distribution becomes wider) in the phase transition domain, while it becomes positive again as the phase transition completes. The minimum of kurtosis is at 80 % HM volume abundance. Notice that the kurtosis is increasing much sharper on the hadronic side, which is a clear consequence of the energy difference between the two phases. This asymmetry appears as a result of the Laurent series expansion, and it would not show up with the usual 4th order polynomial approximation!

In a multi-module or hybrid-model construction (e.g where the PIC hydro stage is attached to a parton and hadron cascade model PACIAE), the flow features are matched [15] to a subsequent dynamical model, which describes dynamical, non-equilibrium, rapid hadronization. This type of models can describe realistically the statistical properties in a dynamical phase transition, which determine the hadron distribution in the final stage.

This stage would then explicitly describe the random fluctuations arising both from the phase transition and the flow dynamics.

In ref. [16] a mixed particle method is introduced, which could separate the fluctuations arising from local critical fluctuations. The "mixed events" are actually eliminating two particle correlations, and only the single particle distributions remain. Thus for central events these are mainly local correlations, which may arise from local fluctuations caused by energy and baryon charge clustering in a phase transition. The method separates the consequences of such correlations.

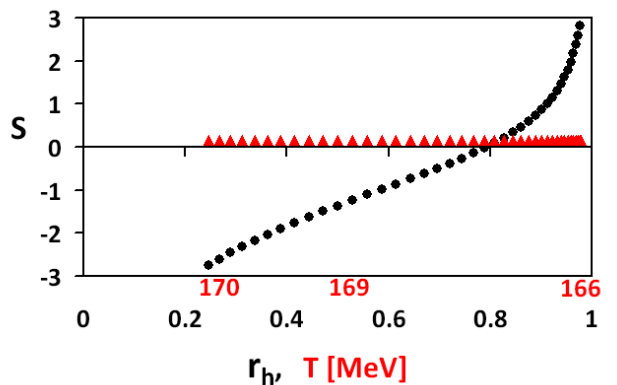


FIG. 4: (color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as  $r_h$ , where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in  $T$ . Results for  $\Omega = 500 fm^3$ .

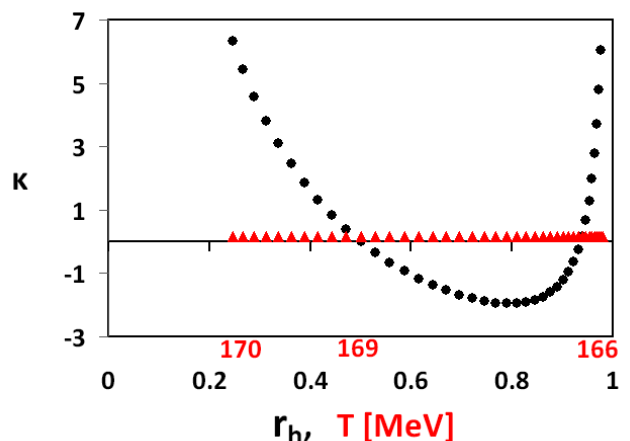


FIG. 5: (color online) Kurtosis as a function of the volume abundance of the hadronic matter (denoted as  $r_h$ , where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in  $T$ . Results for  $\Omega = 500 fm^3$ .

This method can be used both in hybrid model calculations and in experiments, to separate the fluctuation effects from the collective flow and the phase transition dynamics.

## V. CONCLUSIONS

In this work we reiterated earlier results on critical fluctuations [4] in the phase transition between quark-gluon plasma and hadronic matter. This model is specific because of the large difference of the energy density

of the two phases, where  $e_q \gg e_h$ . Here this work is extended to the evaluation of frequently used statistical parameters like the Skewness and Kurtosis of typical parameters like charged particle multiplicities. In continuum models the energy density fluctuations or baryon charge fluctuations could carry the same role, however at ultra relativistic energies a large number of antiparticles are created therefore the energy density fluctuations are better representing the total charge particle multiplicity fluctuation.

Our model is furthermore extended to study the dynamical change of these typical parameters during the hadronization process, where the development is very characteristic and informative for the phase transition we study.

In experiments one can measure these parameters (together with all other measurable quantities) at the Freeze Out (FO) time (or at the FO hypersurface). Luckily the recent RHIC Beam Energy Scan program, scans the same

statistical parameters at a series of different beam energies, which also provide a set of different FO points mapping the dynamics of the phase transition or the most interesting part of it. The FO point at high energies where QGP is formed may be simultaneous to the phase transition, as hadronization must always take place in these reactions. At lower beam energies when QGP is not formed the FO is in the hadronic phase. At very high energies where high energy density QGP is formed in a large volume, the system may have sufficiently large volume and large energy density that FO happens later then the hadronization, i.e. in the pure hadronic state.

The dynamical changes of the statistical parameters this way can provide valuable information at what stage of the phase transition we are at a given reaction.

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